CUBO A Mathematical Journal
Vol.14, N$^{\underline{o}}$03, (59–61). October 2012

Erratum to "on the group of strong symplectic homeomorphisms"

Augustin Banyaga
Department of Mathematics,
The Pennsylvania State University,
University Park, PA 16802.
email: `banyaga@math.psu.edu, augustinbanyaga@gmail.com`

ABSTRACT

We give a proof of the estimate (1.1) which is the main ingredient in the proof that the set $\mathsf{SSympeo}(M, \omega)$ of strong symplectic homeomorphisms of a compact symplectic manifold (M, ω) forms a group [1].

RESUMEN

Probamos la estimación (1.1) que es el principal elemento en la demostración que el conjuntos $\mathsf{SSympeo}(M, \omega)$ de homeomorfismos simplécticos fuertes de una variedad simpléctica compacta (M, ω) genera un grupo [1].

Keywords and Phrases: C^0-symplectic topology; Strong symplectic homeomorphism.

2010 AMS Mathematics Subject Classification: 53D05; 53D35.

1 Erratum

In the paper [1] mentioned in the title, the "constant E" at page 60 may be infinite (so proposition 2 is meaningless). Therefore, some of the estimates on pages 63 to 65 based on E, needed to show that

$$\int_0^1 \mathrm{osc}(v_t^n - v_t^m) \to 0 \tag{1.1}$$

as $n, m \to \infty$ may not hold true. Here is a direct proof of (1.1).

First simplify the notations by writing kH^m for $(\mu_t^k)^* \mathcal{H}^m$, H for $\mathcal{H}$ and omitting t. The function $v^n := v_t^n$ satisfies $nH^n - H^n = dv^n$. Fix a point $*$ in M and for each $x \in M$, pick an arbitrary curve γ_x from $*$ to x, then

$$u^n(x) := \int_{\gamma_x} (nH^n - H^n) = v^n(x) - v^n(*).$$

(The definition of $u^n(x)$ is independent of the choice of the curve γ_x). Hence $\mathrm{osc}(u^n - u^m) = \mathrm{osc}(v^n - v^m)$. Since $\mathrm{osc}(f) \leq 2|f|$, where $|.|$ is the uniform sup norm, we need to show that

$$\begin{aligned}\int_0^1 |\int_{\gamma_x} (nH^n - H^n) - (mH^m - H^m)|dt \leq \int_0^1 |\int_{\gamma_x} (nH^n - mH^m)|dt \\ + \int_0^1 |\int_{\gamma_x} (H^n - H^m)|dt,\end{aligned} \tag{1.2}$$

goes to zero , when n, m are sufficiently large.

The last integral tends to zero when n, m are large: indeed,

$$\begin{aligned}\int_0^1 |\int_{\gamma_x} (H^n - H^m)|dt = \int_0^1 |\int_0^1 (H^n - H^m)_{(\gamma_x(u))}(\gamma_x'(u)du)|dt| \\ \leq A \int_0^1 |H^n - H^m|dt,\end{aligned} \tag{1.3}$$

where $A = \sup_u |\gamma_x'(u)|$. This goes to 0 since H^n is a Cauchy sequence.

To prove that $\int_0^1 |\int_{\gamma_x} (nH^n - mH^m)|dt$ tends to zero when $n, m \to \infty$, we write:

$$\begin{aligned}|\int_{\gamma_x} (nH^n - mH^m)| \leq |\int_{\gamma_x} (nH^n - mH^n)| \\ + |\int_{\gamma_x} (m(H^n - H^m) - n_0(H^n - H^m))| \\ + |\int_{\gamma_x} (n_0)(H^n - H^m)|,\end{aligned} \tag{1.4}$$

for some large n_0.

The integral

$$\int_0^1 |\int_{\gamma_x}(n_0)(H^n - H^m)|dt = \int_0^1 |\int_0^1 (H^n - H^m)_{(\gamma_{n_0}(u))}(D\mu^{n_0}\gamma_x'(u)du)|dt|$$
$$\leq B\int_0^1 |H^n - H^m|dt, \tag{1.5}$$

where $B = \sup_u |D\mu^{n_0}\gamma_x'(u)|$ goes to zero when $n, m \to \infty$ since H^n is a Cauchy sequence and $D\mu^{n_0}$ is bounded. (Here $\gamma_k = \mu^k(\gamma_x)$).

We now show that $\int_{\gamma_x}(nH^n - mH^n) = \int_{\gamma_n} H^n - \int_{\gamma_m} H^n$ tends to zero when $n, m \to \infty$

Let d_0 be a distance induced by a Riemmanian metric g and let r be its injectivity radius. For n, m large enough, $\sup_x d_0(\mu_t^n(x), \mu_t^m(x)) \leq r/2$. It follows that there is a homotopy $F : [0,1] \times M \to M$ between μ^n and μ^m , i.e $F(0,y) = \mu^n(y)$ and $F(1,y) = \mu^m(y)$ and we may define $F(s,y)$ to be the unique minimal geodesic $\nu_{mn}^y(s)$ joining $\mu^n(y)$ to $\mu^m(y)$. See [[3]] (Theorem 12.9). Let $\square(s,u) =: \{F(s,\gamma_n(u)), 0 \leq s,u \leq 1\}$

Since by Stokes' theorem, $\int_{\partial\square} H^n = 0$, $\int_{\gamma_n} H^n - \int_{\gamma_m} H^n = \int_L H^n - \int_{L'} H^n$ where L, and L' are the geodesics ν_{mn}^x and ν_{mn}^*. The integral over L is bounded by $\sup_s |H^n(\nu_{mn}^x(s)| d_0(\mu_t^n(x), \mu_t^m(x))$, because the speed of the geodesics L, L' is bounded by $d_0(\mu_t^n(x), \mu_t^m(x))$. This integral tends to zero when $n, m \to \infty$ since H^n is also bounded . Analogously for the integral over L'.

The same argument apply to $H^n - H^m$ with the geodesics L, L' replaced by $\nu_{mn_0}^x$ and $\nu_{mn_0}^*$. This finishes the proof of (1.1).

Remark : We will show in a forthcoming paper [2] that (1.1) is the main ingredient in the proof of the main theorem of [1].

Acknowledgments

I would like to thank Mike Usher to have pointed out to me that Proposition 2 in [1] does not yield a finite constant E.

Received: January 2012. Revised: January 2012.

References

[1] A. Banyaga, On the group of strong symplectic homeomorphisms, Cubo, Vol 12, No 03 (2010), 49-69

[2] A. Banyaga, On the group of strong symplectic homeomorphisms, II, preprint

[3] T. Brocker and K. Janich, Introduction to differential topology, Cambrigde University Press, 1982