CUBO A Mathematical Journal
Vol.13, $N^{\underline{o}}$01, (137–147). March 2011

Strong convergence of an implicit iteration process for a finite family of strictly asymptotically pseudocontractive mappings

Gurucharan Singh Saluja
Department of Mathematics & Information Technology,
Govt. Nagarjuna P.G. College of Science, Raipur (C.G.).
email: saluja_1963@rediffmail.com
and
Hemant Kumar Nashine
Department of Mathematics,
Disha Institute of Management and Technology,
Satya Vihar, Vidhansabha-Chandrakhuri Marg
Mandir Hasaud, Raipur-492101(Chhattisgarh), India.
email: hnashine@rediffmail.com, nashine_09@rediffmail.com

ABSTRACT

In this paper, we establish the strong convergence theorems for a finite family of k-strictly asymptotically pseudo-contractive mappings in the framework of Hilbert spaces. Our results improve and extend the corresponding results of Liu [5] and many others.

RESUMEN

En este trabajo, hemos establecido los teoremas de convergencia para una familia finita de asignaciones de k-estrictamente asintticamente pseudo-contraccin en el marco de los espacios de Hilbert. Nuestros resultados mejoran y amplan los resultados correspondientes de Liu [5] y muchos otros.

Keywords: Strictly asymptotically pseudo-contractive mapping, implicit iteration scheme, common fixed point, strong convergence, Hilbert space.

AMS Subject Classification: 47H09, 47H10.

1 Introduction

Let H be a real Hilbert space with the scalar product and norm denoted by the symbols $\langle .,. \rangle$ and $\| . \|$ respectively, and C be a closed convex subset of H. Let T be a (possibly) nonlinear mapping from C into C. We now consider the following classes:

(1) T is contractive, i.e., there exists a constant $k < 1$ such that

$$\|Tx - Ty\| \quad \leq \quad k \|x - y\|, \tag{1.1}$$

for all $x, y \in C$.

(2) T is nonexpansive, i.e.,

$$\|Tx - Ty\| \quad \leq \quad \|x - y\|, \tag{1.2}$$

for all $x, y \in C$.

(3) T is uniformly L-Lipschitzian, i.e., if there exists a constant $L > 0$ such that

$$\|T^n x - T^n y\| \quad \leq \quad L \|x - y\|, \tag{1.3}$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

(4) T is pseudo-contractive, i.e.,

$$\langle Tx - Ty, j(x - y) \rangle \quad \leq \quad \|x - y\|^2, \tag{1.4}$$

for all $x, y \in C$.

(5) T is strictly pseudo-contractive, i.e., there exists a constant $k \in [0, 1)$ such that

$$\|Tx - Ty\|^2 \quad \leq \quad \|x - y\|^2 + k \|(x - Tx) - (y - Ty)\|^2, \tag{1.5}$$

for all $x, y \in C$.

(6) T is asymptotically nonexpansive [3], i.e., if there exists a sequence $\{r_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} r_n = 0$ such that

$$\|T^n x - T^n y\| \leq (1 + r_n) \|x - y\|, \tag{1.6}$$

for all $x, y \in C$ and $n \in \mathbb{N}$.

(7) T is k-strictly asymptotically pseudo-contractive [6], i.e., if there exists a sequence $\{r_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} r_n = 0$ such that

$$\begin{aligned} \|T^n x - T^n y\|^2 \leq\ & (1 + r_n)^2 \|x - y\|^2 \\ & + k \|(x - T^n x) - (y - T^n y)\|^2 \end{aligned} \tag{1.7}$$

for some $k \in [0, 1)$ for all $x, y \in C$ and $n \in \mathbb{N}$.

Remark 1.1 [6]: If T is k-strictly asymptotically pseudo-contractive mapping, then it is uniformly L-Lipschitzian, but the converse does not hold.

Concerning the convergence problem of iterative sequences for strictly pseudocontractive mappings has been studied by several authors (see, e.g., [2, 4, 7, 11, 12]). Concerning the class of strictly asymptotically pseudocontractive mappings, Liu [5] proved the following result in Hilbert space:

Theorem 1.1(Liu [5]): Let H be a real Hilbert space, let C be a nonempty closed convex and bounded subset of H, and let $T\colon C \to C$ be a completely continuous uniformly L-Lipschitzian $(\lambda, \{k_n\})$-strictly asymptotically pseudocontractive mapping such that $\sum_{n=1}^{\infty}(k_n^2 - 1) < \infty$. Let $\{\alpha_n\} \subset (0, 1)$ be a sequence satisfying the following condition:

$$0 < \epsilon \leq \alpha_n \leq 1 - \lambda - \epsilon \quad \forall\ n \geq 1 \text{ and some } \epsilon > 0.$$

Then, the sequence $\{x_n\}$ generated from an arbitrary $x_1 \in C$ by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T^n x_n, \quad \forall\ n \geq 1 \tag{1.8}$$

converges strongly to a fixed point of T.

In 2001, Xu and Ori [12] have introduced an implicit iteration process for a finite family of nonexpansive mappings in a Hilbert space H. Let C be a nonempty subset of H. Let $T_1, T_2, \ldots, T_N$ be self-mappings of C and suppose that $\mathcal{F} = \cap_{i=1}^{N} F(T_i) \neq \emptyset$, the set of common fixed points of $T_i, i = 1, 2, \ldots, N$. An implicit iteration process for a finite family of nonexpansive mappings is defined as follows, with $\{t_n\}$ a real sequence in $(0, 1)$, $x_0 \in C$:

$$\begin{aligned}
x_1 &= t_1x_0 + (1-t_1)T_1x_1, \\
x_2 &= t_2x_1 + (1-t_2)T_2x_2, \\
&\vdots \\
x_N &= t_Nx_{N-1} + (1-t_N)T_Nx_N, \\
x_{N+1} &= t_{N+1}x_N + (1-t_{N+1})T_1x_{N+1}, \\
&\vdots
\end{aligned}$$

which can be written in the following compact form:

$$x_n = t_nx_{n-1} + (1-t_n)T_nx_n, \quad n \geq 1 \tag{1.9}$$

where $T_k = T_{k \bmod N}$. (Here the mod N function takes values in $\{1, 2, \ldots, N\}$). And they proved the weak convergence of the process (1.9).

Very recently, Acedo and Xu [1] still in the framework of Hilbert spaces introduced the following cyclic algorithm.

Let C be a closed convex subset of a Hilbert space H and let $\{T_i\}_{i=0}^{N-1}$ be N k-strict pseudo-contractions on C such that $F = \bigcap_{i=0}^{N-1} F(T_i) \neq \emptyset$. Let $x_0 \in C$ and let $\{\alpha_n\}$ be a sequence in $(0, 1)$. The cyclic algorithm generates a sequence $\{x_n\}_{n=1}^{\infty}$ in the following way:

$$\begin{aligned}
x_1 &= \alpha_0x_0 + (1-\alpha_0)T_0x_0, \\
x_2 &= \alpha_1x_1 + (1-\alpha_1)T_1x_1, \\
&\vdots \\
x_N &= \alpha_{N-1}x_{N-1} + (1-\alpha_{N-1})T_{N-1}x_{N-1}, \\
x_{N+1} &= \alpha_Nx_N + (1-\alpha_N)T_0x_N, \\
&\vdots
\end{aligned}$$

In general, $\{x_{n+1}\}$ is defined by

$$x_{n+1} = \alpha_nx_n + (1-\alpha_n)T_{[n]}x_n, \tag{1.10}$$

where $T_{[n]} = T_i$ with $i = n \ (mod\ N)$, $0 \le i \le N-1$. They also proved a weak convergence theorem for k-strict pseudo-contractions in Hilbert spaces by cyclic algorithm (1.10). More precisely, they obtained the following theorem:

Theorem AX [1]: Let C be a closed convex subset of a Hilbert space H. Let $N \ge 1$ be an integer. Let for each $0 \le i \le N-1$, $T_i \colon C \to C$ be a k_i-strict pseudo-contraction for some $0 \le k_i < 1$. Let $k = \max\{k_i : 1 \le i \le N\}$. Assume the common fixed point the set $\bigcap_{i=0}^{N-1} F(T_i)$ of $\{T_i\}_{i=0}^{N-1}$ is nonempty. Given $x_0 \in C$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by the cyclic algorithm (1.10). Assume that the control sequence $\{\alpha_n\}$ is chosen so that $k + \epsilon < \alpha_n < 1 - \epsilon$ for all n and for some $\epsilon \in (0,1)$. Then $\{x_n\}$ converges weakly to a common fixed point of the family $\{T_i\}_{i=0}^{N-1}$.

Motivated by Xu and Ori [12], Acedo and Xu [1] and some others we introduce and study the following:

Let C be a closed convex subset of a Hilbert space H and let $\{T_i\}_{i=0}^{N-1}$ be N k-strictly asymptotically pseudo-contractions on C such that $F = \bigcap_{i=0}^{N-1} F(T_i) \ne \emptyset$. Let $x_0 \in C$ and let $\{\alpha_n\}$ be a sequence in $(0,1)$. The implicit iteration scheme generates a sequence $\{x_n\}_{n=0}^{\infty}$ in the following way:

$$
\begin{aligned}
x_1 &= \alpha_0 x_0 + (1-\alpha_0) T_0 x_0,\\
x_2 &= \alpha_1 x_1 + (1-\alpha_1) T_1 x_1,\\
&\vdots\\
x_N &= \alpha_{N-1} x_{N-1} + (1-\alpha_{N-1}) T_{N-1} x_{N-1},\\
x_{N+1} &= \alpha_N x_N + (1-\alpha_N) T_0^2 x_0,\\
&\vdots\\
x_{2N} &= \alpha_{2N-1} x_{2N-1} + (1-\alpha_{2N-1}) T_{N-1}^2 x_{2N-1},\\
x_{2N+1} &= \alpha_{2N} x_{2N} + (1-\alpha_{2N}) T_0^3 x_0,\\
&\vdots
\end{aligned}
$$

In general, $\{x_n\}$ is defined by

$$
x_{n+1} = \alpha_n x_n + (1-\alpha_n) T_{[n]}^s x_n, \tag{1.11}
$$

where $T_{[n]}^s = T_{n\,(mod\ N)}^s = T_i^s$ with $n = (s-1)N + i$ and $i \in I = \{0, 1, \ldots, N-1\}$.

The aim of this paper is to establish strong convergence theorems of implicit iteration process (1.11) for a finite family of k-strictly asymptotically pseudo-contraction mappings in Hilbert spaces. Our results extend the corresponding results of Liu [5] and many others.

In the sequel, we will need the following lemmas.

Lemma 1.1: Let H be a real Hilbert space. There holds the following identities:

(i) $\|x-y\|^2 = \|x\|^2 - \|y\|^2 - 2\langle x-y, y\rangle \quad \forall\ x, y \in H.$

(ii) $\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2,$

$\forall\ t \in [0,1],\ \forall\ x, y \in H.$

(iii) If $\{x_n\}$ be a sequence in H weakly converges to z, then

$$\limsup_{n\to\infty} \|x_n - y\|^2 = \limsup_{n\to\infty} \|x_n - z\|^2 + \|z-y\|^2 \quad \forall y \in H.$$

Lemma 1.2 [9]: Let $\{a_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{r_n\}_{n=1}^{\infty}$ be sequences of nonnegative real numbers satisfying the inequality

$$a_{n+1} \le (1+r_n)a_n + \beta_n, \quad n \ge 1.$$

If $\sum_{n=1}^{\infty} r_n < \infty$ and $\sum_{n=1}^{\infty} \beta_n < \infty$, then $\lim_{n\to\infty} a_n$ exists. If in addition $\{a_n\}_{n=1}^{\infty}$ has a subsequence which converges strongly to zero, then $\lim_{n\to\infty} a_n = 0$.

2 Main Results

Theorem 2.1: Let C be a closed convex subset of a Hilbert space H. Let $N \ge 1$ be an integer. Let for each $0 \le i \le N-1$, $T_i\colon C \to C$ be N k_i-strictly asymptotically pseudo-contraction mappings for some $0 \le k_i < 1$ and $\sum_{n=1}^{\infty} r_n < \infty$. Let $k = \max\{k_i : 0 \le i \le N-1\}$ and $r_n = \max\{r_{n_i} : 0 \le i \le N-1\}$. Assume that $F = \bigcap_{i=0}^{N-1} F(T_i) \ne \emptyset$. Given $x_0 \in C$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by an implicit iteration scheme (1.11). Assume that the control sequence $\{\alpha_n\}$ is chosen so that $k < \alpha_n < 1$ for all n and $\sum_{n=0}^{\infty}(\alpha_n - k)(1-\alpha_n) = \infty$. Then the iterative sequence $\{x_n\}$ has the following properties:

(1) $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F$,

(2) $\lim_{n\to\infty} d(x_n, F)$ exists,

(3) $\liminf_{n\to\infty} \left\|x_n - T^s_{[n]}x_n\right\| = 0$,

(4) the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to a common fixed point $p \in F$ if and only if

$$\liminf_{n\to\infty} d(x_n, F) = 0.$$

Proof: We divide the proof of Theorem 2.1 into three steps.

(I) First, we proof the conclusions (1)and (2).

For any $p \in F$, it follows from (1.11) and Lemma 1.1(ii), we note that

$$\begin{aligned}
\|x_{n+1} - p\|^2 &= \left\|\alpha_n x_n + (1-\alpha_n)T^s_{[n]}x_n - p\right\| \\
&= \left\|\alpha_n(x_n - p) + (1-\alpha_n)(T^s_{[n]}x_n - p)\right\| \\
&\le \alpha_n \|x_n - p\|^2 + (1-\alpha_n)\left\|T^s_{[n]}x_n - p\right\|^2 \\
&\quad -\alpha_n(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2 \\
&\le \alpha_n \|x_n - p\|^2 + (1-\alpha_n)[(1+r_n)^2\|x_n - p\|^2 \\
&\quad +k\left\|x_n - T^s_{[n]}x_n\right\|^2] - \alpha_n(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2 \\
&\le [\alpha_n(1+r_n)^2 + (1-\alpha_n)(1+r_n)^2]\|x_n - p\|^2 \\
&\quad -(\alpha_n - k)(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2 \\
&\le (1+r_n)^2\|x_n - p\|^2 - (\alpha_n - k)(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2 \\
&\le (1+d_n)\|x_n - p\|^2 - (\alpha_n - k)(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2
\end{aligned} \tag{2.1}$$

where $d_n = r_n^2 + 2r_n$, since $\sum_{n=1}^{\infty} r_n < \infty$ thus $\sum_{n=1}^{\infty} d_n < \infty$ and since $k < \alpha_n < 1$, we get

$$\|x_{n+1} - p\|^2 \le (1+d_n)\|x_n - p\|^2 \tag{2.2}$$

and therefore

$$\|x_{n+1}-p\| \leq (1+d_n)^{1/2}\|x_n-p\|. \tag{2.3}$$

Since $\sum_{n=1}^{\infty} d_n < \infty$, it follows from Lemma 1.2, we know that $\lim_{n\to\infty}\|x_n-p\|$ exists for each $p \in F$. So that there exists $K > 0$ such that $\|x_n-p\| \leq K$ for all $n \geq 1$. Consequently, we obtain from (2.3) that

$$\begin{aligned}\|x_{n+1}-p\| &\leq (1+d_n)^{1/2}\|x_n-p\| \\ &\leq (1+d_n)\|x_n-p\| \\ &\leq \|x_n-p\| + Kd_n. \end{aligned} \tag{2.4}$$

It follows from (2.4) that

$$d(x_{n+1},F) \leq (1+d_n)d(x_n,F), \quad \forall\ n \geq 1 \tag{2.5}$$

so that it again follows from Lemma 1.2 that $\lim_{n\to\infty} d(x_n,F)$ exists.
The conclusions (1)and (2) are proved.

(II) The proof of conclusion (3).

It follows from (2.1) that

$$\begin{aligned}\|x_{n+1}-p\|^2 &\leq (1+d_n)\|x_n-p\|^2 \\ &\quad -(\alpha_n-k)(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2\end{aligned} \tag{2.6}$$

where $d_n = r_n^2 + 2r_n$, since $\sum_{n=1}^{\infty} r_n < \infty$ thus $\sum_{n=1}^{\infty} d_n < \infty$ and since $k < \alpha_n < 1$, we get

$$\|x_{n+1}-p\|^2 \leq (1+d_n)\|x_n-p\|^2 \tag{2.7}$$

that means the sequence $\{\|x_n-p\|\}$ is decreasing. Now, since $\sum_{n=1}^{\infty} d_n < \infty$ it follows that $\prod_{i=1}^{\infty}(1+d_i) < \infty$, from (2.6), we have

$$\begin{aligned}\sum_{n=0}^{\infty}(\alpha_n-k)(1-\alpha_n)\left\|x_n - T^s_{[n]}x_n\right\|^2 &\leq \prod_{i=1}^{\infty}(1+d_i)\|x_0-p\|^2 \\ &< \infty.\end{aligned} \tag{2.8}$$

Since $\sum_{n=0}^{\infty}(\alpha_n - k)(1-\alpha_n) = \infty$, (2.8) implies that

$$\liminf_{n\to\infty} \left\| x_n - T_{[n]}^s x_n \right\| = 0. \tag{2.9}$$

(IV) Next, we prove the conclusion (4).

Necessity

If $\{x_n\}$ converges strongly to some point $p \in F$, then from $0 \leq d(x_n, F) \leq \|x_n - p\| \to 0$ as $n \to \infty$, we have

$$\liminf_{n\to\infty} d(x_n, F) = 0. \tag{2.10}$$

Sufficiency

If $\liminf_{n\to\infty} d(x_n, F) = 0$, it follows from the conclusion (2) that $\lim_{n\to\infty} d(x_n, F) = 0$. Next, we prove that $\{x_n\}$ is a Cauchy sequence in C. In fact, since for any $x > 0$, $1 + x \leq exp(x)$, therefore, for any $m, n \geq 1$ and for given $p \in F$, from (2.4), we have

$$\begin{aligned} \|x_{n+m} - p\| &\leq (1 + d_{n+m-1}) \|x_{n+m-1} - p\| \\ &\leq e^{d_{n+m-1}} \|x_{n+m-1} - p\| \\ &\leq e^{d_{n+m-1}} [e^{d_{n+m-2}} \|x_{n+m-2} - p\|] \\ &\leq e^{\{d_{n+m-1} + d_{n+m-2}\}} \|x_{n+m-2} - p\| \\ &\leq \ldots \\ &\leq e^{\sum_{j=n}^{n+m-1} d_j} \|x_n - p\| \\ &\leq K' \|x_n - p\| < \infty \end{aligned} \tag{2.11}$$

where $K' = e^{\sum_{j=1}^{\infty} d_j} < \infty$. Since

$$\lim_{n\to\infty} d(x_n, F) = 0, \tag{2.12}$$

for any given $\epsilon > 0$, there exists a positive integer n_1 such that

$$d(x_n, F) \quad < \quad \frac{\epsilon}{2(K'+1)} \quad , \ \forall \ n \geq n_1. \tag{2.13}$$

Hence, there exists $p_1 \in F$ such that

$$\|x_n - p_1\| \quad < \quad \frac{\epsilon}{(K'+1)} \quad \forall \ n \geq n_1. \tag{2.14}$$

Consequently, for any $n \geq n_1$ and $m \geq 1$, from (2.11), we have

$$\begin{aligned} \|x_{n+m} - x_n\| &\leq \|x_{n+m} - p_1\| + \|x_n - p_1\| \\ &\leq K' \|x_n - p_1\| + \|x_n - p_1\| \\ &\leq (K'+1) \|x_n - p_1\| \\ &< (K'+1) . \frac{\epsilon}{(K'+1)} = \epsilon. \end{aligned}$$

This implies that $\{x_n\}$ is a Cauchy sequence in C. Let $x_n \to x^* \in C$. Since $\liminf_{n\to\infty} d(x_n, F) = 0$, and so $d(x^*, F) = 0$. Again since $\{T_i\}_{i=0}^{N-1}$ is a finite family of k-strictly asymptotically pseudo-contractive mappings, by Remark 1.1 of [6], it is a finite family of uniformly Lipschitzian mappings. Hence, the set F of common fixed points of $\{T_i\}_{i=0}^{N-1}$ is closed and so $x^* \in F$. Thus the sequence $\{x_n\}$ converges strongly to a common fixed point of the family $\{T_i\}_{i=0}^{N-1}$. This completes the proof.

Theorem 2.2: Let C be a closed convex compact subset of a Hilbert space H. Let $N \geq 1$ be an integer. Let for each $0 \leq i \leq N-1$, $T_i \colon C \to C$ be N k_i-strictly asymptotically pseudo-contraction mappings for some $0 \leq k_i < 1$ and $\sum_{n=1}^{\infty} r_n < \infty$. Let $k = \max\{k_i : 0 \leq i \leq N-1\}$ and $r_n = \max\{r_{n_i} : 0 \leq i \leq N-1\}$. Assume that $F = \bigcap_{i=0}^{N-1} F(T_i) \neq \emptyset$. Given $x_0 \in C$, let $\{x_n\}_{n=0}^{\infty}$ be the sequence generated by an implicit iteration scheme (1.11). Assume that the control sequence $\{\alpha_n\}$ is chosen so that $k < \alpha_n < 1$ for all n. Then $\{x_n\}$ converges strongly to a common fixed point of the family $\{T_i\}_{i=0}^{N-1}$.

Proof: We only conclude the difference. By compactness of C this immediately implies that there is a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ which converges to a common fixed point of $\{T_i\}_{i=0}^{N-1}$, say, p. Combining (2.3) with Lemma 1.2, we have $\lim_{n\to\infty} \|x_n - p\| = 0$. Thus $\{x_n\}$ converges strongly to a common fixed point of the family $\{T_i\}_{i=0}^{N-1}$. This completes the proof.

Remark 2.1 Our results extend and improve the corresponding results of Liu [5] and we also extend the iteration process (1.8) of [5] to an implicit iteration process for a finite family of mappings.

Received: June 2009. Revised: November 2009.

References

[1] G.L. Acedo and H.K. Xu, *Iterative methods for strict pseudo-contractions in Hilbert spaces*, Nonlinear Anal. 67(2007), 2258-2271.

[2] F.E. Browder and W.V. Ptryshyn, *Construction of fixed points of nonlinear mappings in Hilbert spaces*, J. Math. Anal. Appl. 20(1967), 197-228.

[3] K. Goebel and W.A. Kirk, *A fixed point theorem for asymptotically nonexpansive mappings*, Proc. Amer. Math. Soc. 35(1972), 171-174.

[4] F. Gu, *The new composite implicit iterative process with errors for common fixed points of a finite family of strictly pseudocontractive mappings*, J. Math. Anal. Appl. 329(2) (2007), 766-776.

[5] Q. Liu, *Convergence theorems of the sequence of iterates for asymptotically demicontractive and hemicontractive mappings*, Nonlinear Anal. 26(1996), 1835-1842.

[6] M.O. Osilike, *Iterative approximation of fixed points of asymptotically demicontractive mappings*, Indian J. Pure Appl. Math. 29(12), December 1998, 1291-1300.

[7] M.O. Osilike, *Implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps*, J. Math. Anal. Appl. 294(1)(2004), 73-81.

[8] M.O. Osilike and A. Udomene, *Demiclosedness principle and convergence results for strictly pseudocontractive mappings of Browder-Petryshyn type*, J. Math. Anal. Appl. 256(2001), 431-445.

[9] M.O. Osilike, S.C. Aniagbosor and B.G. Akuchu, *Fixed points of asymptotically demicontractive mappings in arbitrary Banach spaces*, PanAm. Math. J. 12(2002), 77-78.

[10] S. Reich, *Weak convergence theorems for nonexpansive mappings in Banach spaces*, J. Math. Anal. Appl. 67(1979), 274-276.

[11] Y. Su and S. Li, *Composite implicit iteration process for common fixed points of a finite family of strictly pseudocontractive maps*, J. Math. Anal. Appl. 320(2)(2006), 882-891.

[12] H.K. Xu and R.G. Ori, *An implicit iteration process for nonexpansive mappings*, Numer. Funct. Anal. Optim. 22(2001), 767-773.