Cubo Matemática Educacional
Vol., 4. N°2, Junio 2002

THE TRANSMISSION OF GREEK GEOMETRY TO MEDIEVAL ISLAM

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Although the mathematical sciences in medieval Islam developed during the eighth and ninth centuries on the basis of material from Sassanian Iran, India, and Greece, the primary influence was Greek. Whether one judges by the amount of material translated or its significance for medieval Islamic mathematics, the Greek heritage far outweighed the others and was the primary determinant for the direction mathematics took in Islamic civilization. And when we study the transmission of Greek mathematics to medieval Islam there are at least three questions we must ask:

1. What was translated, when and by whom?
2. Why was it translated?
3. What difference did it make?

The first question focuses mainly on Greek mathematics and its translators. But the last two questions focus on medieval Islam and take account of the fact that translation was a complex process, which began rather than ended with the translation of relevant books. Byzantium had more of Archimedes' works, for example, than Islam did, yet did far less with them. One culture acquires ideas methods and techniques from another because people engage themselves with them. They study books. They consult tables. They use instruments. And they do so for reasons that are in part unique to them and in part a reflect interests and concerns they share with society as a whole. It is in this spirit we turn to the Islamic acquisition of Greek science.

There are two things to be aware of in studying medieval Islam's reception of Greek mathematics.

1. The term "medieval Islam" is an abstraction. It is a useful abstraction, to be sure, but it is useful only when we are aware that it is an abstraction. The reality is not a phrase but a complex historical process that extended geographically from Andalusia to Afghanistan and in time over a number of centuries from the eighth century onward. It was a process in which many factors operated.
2. We evaluate the transmission from the present and our task must be to try to imagine what it meant to the people of that time, not to evaluate the process in terms of the present.

One effect of focusing overly on the present is to treat medieval Islam only as a conduit through which Greek mathematics flowed to the Renaissance. Although it was a conduit, and this paper concerns some of the goods it carried, it was more than that. It shaped the material it acquired for its own ends and it developed new material of its own, both extending what it had acquired and developing it in entirely new areas.
Another effect is reading Islamic mathematics as prefiguring modern mathematics. Thus, various writers profess to find intimations of such later innovations as analytic (or even algebraic) geometry, the calculus, four dimensional space, and non-euclidean geometry.
Curiously, both of these errors involve similar attitudes towards medieval Islamic mathematics. In the first case it is valued because of what it did for us, and in the second it is valued to the extent that it resembles us. Our approach here will avoid both of these pitfalls and will focus on medieval Islam and how and why it acquired Greek mathematics.
Knowledge, in what became the standard medieval Islamic classification of sciences, was of two kinds: religious and foreign (or 'sciences of the ancients' as they were also called). The religious sciences were those subjects concerned with the faith and community of Islam. These included Arabic grammar, the correct reading of the Qur'an, the study of the sayings of Muhammad and his companions, and religious law. The foreign sciences, on the other hand, encompassed knowledge acquired from ancient, non-Islamic sources, and included medicine, mathematics, astronomy, and philosophy.
This division of knowledge had important consequences, and some scholars have argued that they were largely negative. Thus, Itzak Goldziher, fol-

lowing the thirteenth-century geographer and biographer Ya'qut wrote that "As soon as someone displayed an interest in the 'ulum al-awa'il [sciences of the ancients] he was regarded as suspect." Even mathematics could be suspect, since it was closely linked to astronomy, and that to astrology. From there on it was downhill into a world of planetary deities and magic. This association could have been one of the causes for the complaint of the tenth-century geometer, al-Sijzi, that where he lived "the great mass of people consider the investigation of geometry blasphemous and ... find it lawful to kill him who believes in its correctness with perseverance ....".

However, the whole question is much more complex than Goldziher portrayed it, and other segments of orthodox opinion taught that it was a matter of balance and judgement. As the distinguished 13th-century historian, Ibn Khaldun put it, "In this respect the sin falls on the sinner," i.e. the individual not the science was to blame if foreign knowledge led a person into heresy.

The Reception of Greek mathematics

We have already said that Greek mathematics became the determining external factor in the mathematical sciences in medieval Islam. Perhaps one reason is that Muslim intellectuals were in close contact from the very earliest times with the direct heirs to Greek learning in the Byzantine lands that Islam conquered.

There was a learned environment in which there were scholars familiar with names like Euclid, Archimedes, and Ptolemy and who could interpret the Greek philosophical tradition. Moreover, they knew how to edit and translate Greek texts, having already translated them into Syriac - a language related to Arabic. These scholars may have been Christian, Jewish, or of other faiths, but they participated enthusiastically in Islam's search for foreign learning. For example, the famous Christian translator, Hunayn ibn Ishaq, writes, of his search for the book *De demonstratione* by the Greek medical writer, Galen [quoted from Berggren 1986, p.4]. He tells how his friend Gabriel first looked for it and then says, "I myself searched with great zeal for this book over Mesopotamia, all of Syria, in Palestine, and Egypt until I came to Alexandria. I found nothing except in Damascus, about half of it. But what I found was neither successive chapters, nor complete. However, Gabriel found some chapters of this book, which are not the same as those I found".

Moreover, both the State and wealthy families actively supported the translation and diffusion of foreign science. For example, the Caliph al-Ma'mun (who reigned from 813 to 833) founded a research and teaching centre in Baghdad known as The House of Wisdom. Members of wealthy families, such as the Banu Musa, themselves travelled to Byzantine lands to purchase mathematical texts for translation. These three brothers were educated in the House of Wisdom which, their father's friend, the caliph al-Ma'mun had established in Baghdad in the early ninth century.

Not all of the reception of foreign science was welcoming, however, and there were critical reactions to it. There is, for example, the story of a Muslim theologian who criticized mathematics on account of the rigidity of the subject. Specifically he objected that in mathematics:

1. Some words had no meaning given to them.
2. Other words had only one meaning, with no possibility of modification when new relationships involving the concept were discovered.
3. Its arguments were limited to deductive logic.

These are clearly arguments of someone who understands what he is criticizing and they exemplify the dialogue that was going on. Criticism of a seemingly different sort was that by the thirteenth century Yemeni scholar, Ibrahim al-Asbahi who wrote a *Book of Pearls on the Science of Timekeeping*. He said, "The times of prayer are to be found by observation with one's eyes. They are not to be found by the markings on the astrolabe or by calculation using the science of the astronomers ... . The astronomers took their knowledge from Euclid, the Indian astronomical tradition recorded by the authors of the Sindhind, as well as Aristotle and other philosophers, and all of them were infidels." However, this remark must be set in the context of the basis of the objection, namely that non-Islamic sources were being used for such essential Islamic activities as the determination of the dates of the new moon, the times of the five daily prayers, and finding the direction of Mecca.

With these remarks as background we turn now to the Islamic acquisition of Greek geometry, a story that begins with Euclid's *Elements*.

Over (roughly) the century from the reign of Harun al-Rashid (786 - 809) to the death of Thabit ibn Qurra in 901 translators produced at least four Arabic translations and a major revision of the *Elements*. Two caliphs, Harun al-Rashid (of the *Thousand and One Nights* fame) and al-Ma'mun supported the work. The details are extremely complicated, however, and I can only refer you to. Brentjes 1996 for a recent account.

As for the motivations for translating the *Elements*, I believe that there were at least three important motivations.

1. Ptolemy's great astronomical work, *The Almagest*, was translated around the same time and certainly a knowledge of the contents of at least the first six books of the *Elements* would have been essential for anyone who wanted to understand *The Almagest*. This connection with astronomy also explains why, with the translation of the *Elements*, there were also translations of a number of the so-called "Middle Books," books studied after the *Elements* but before *The Almagest*. They included Euclid's *Data* and his *Phaenomena*, Theodosius's *Sphaerica*, Menelaus's work of the same title (translated by Hunayn ibn Ishaq (b. 809)), Hypsicles *On Rising Time*, and Aristarchus's *On the Size and Distance of the Sun and Moon*.

2. The translation movement was going on at the same time that there were debates on religion. Caliphs sponsored discussions at court between representatives of different religions, and logic became an important tool for holding one's own in these debates. A work such as the *Elements* which unquestionably trains one in logical thinking must have been at least congenial to such an atmosphere.

3. Finally, Islam had a positive attitude towards knowledge. The Prophet Muhammad said, "Seek knowledge even in China." And although the status of the foreign knowledge represented by a work such as the *Elements* was not always uniformly high, those who attacked it often did so on the basis of overstudy of such works. Excessive study in these areas led to the neglect of a person's obligations to the religious community of Dar al-Islam. Thus the famous physician al-Razi (see Berggren 1996, p. 271), says that he has studied mathematics only to the extent that it is absolutely necessary. He avoided, he tells us, the path of the "so-called philosophers who devote their whole lives to studying geometrical superfluities".

Whatever the motivations may have been, however, the consequences of the acquisition of the *Elements* for mathematics in Islam were immense. Quite apart from furnishing a base of geometrical knowledge that was the stock-in-trade of all subsequent geometers, it furnished a paradigm for what a mathematical argument should be. It thereby ensured that the Greek model of axiom-definition-theorem-proof became the model for Islamic geometry as well.

It also stimulated foundational investigations on the theory of parallels and ratios. Some of the work on parallels found its way into 17th century European treatments of the problem, in particular into the work of John Wallis. The work on ratios produced, in the writings of Omar Khayyam and others, important modifications of the idea of number which included the expansion of that concept to include the ratio of any two magnitudes.

The effects of the *Elements* were not, however, limited to geometry. It also provided the material for a rigorous development of algebra via Thabit ibn Qurra and Omar Khayyam. Thabit used Euclid to give a proof of the validity of the procedure found in al-Khwarizmi's Algebra for the solution of quadratic equations. Omar's work presupposed not only the *Elements* but Euclid's *Data* for his solution of cubic equations.

Not all consequences were positive, however. The appeal of euclidean rigor led, on at least one occasion, to an intellectual rigidity with unfortunate consequences. I have told the details of this elsewhere. Here, I shall mention only that in the tenth century one of the most eminent of Islamic geometers, Abu Sahl al-Kuhi, was led by his investigations into centers of gravity to conclude that p = 3 1/9. When he was challenged that Archimedes, for whom he of all people had the highest regard, had arrived at a result that contradicted his, he pointed out that Archimedes' work was approximate. Such a method of approximation, Abu Sahl said, was unworthy of a geometer, whose goal is exact knowledge and not approximations to the truth. This was one of those cases of rigor leading to *rigor mortis*.

Archimedes

Thabit ibn Qurra translated all of the Arabic works ascribed to Archimedes: *Sphere and Cylinder* , *Measurement of the Circle, Construction of the Heptagon in the Circle*, and *The Lemmas*. There is some evidence, however, that in the case of *Sphere and Cylinder* Thabit reworked an earlier translation by someone who evidently had quite a struggle with the Greek text. According to al-Tusi even Thabit's improved translation lacks some assumptions, which evidently the original translator had not understood and so had left out.

It was no doubt because of such problems understanding a difficult book that Ishaq ibn Hunayn, when he translated *SC*, also translated the commentary of the sixth century Greek commentator, Eutocius of Ascalon.

The efforts of the translators paid real dividends, however, since the works of Archimedes had a profound influence on Islamic mathematics. For example, the translation of Archimedes' work on the construction of the regular heptagon introduced Islamic geometers to techniques for solving construction problems quite different from those of Euclid's *Elements*. That the Archimedean solution demanded more than the straight-edge and compass solutions of the *Elements* challenged Islamic geometers to attempt what Archimedes had failed to do. They succeeded to the extent that they could show how Archimedes' strange construction could be accomplished by the use of conic sections. This success in fitting Archimedes' construction into an established theory no doubt increased their confidence in their abilities to solve difficult problems.

It is quite clear, also, that the Archimedean tradition of geometrical statics survived into the Islamic world, although we do not yet know which Greek works lay at the roots of this tradition. In any case, al-Kuhi (to cite one example) was aware of the connection between finding centers of gravity of plane figures and volumes of revolution, for he speaks of it in his treatise on finding the volume of the paraboloid.

Archimedes work on finding the area of a parabola was not translated into Arabic. However, his solution to the problem itself was known from his statement of the result in his preface to *SC* I. Geometers in medieval Islam then took it as a challenge to find the proof of the result Archimedes stated, and the first to succeed was evidently Thabit ibn Qurra. Some criticized Thabit's solution as being too long, so his grandson, Ibrahim ibn Sinan saved the family's honor by coming up with a more elegant solution. Archimedes' work also inspired Islamic work finding the volumes of figures formed from the rotation of segments of parabolas, both about their axes and about their ordinates to the axes. Among those contributing to this work were Thabit, al-Kuhi and Ibn al-Haytham (the 'Alhazen" of the Latin West).

The second book, *SC* II (particularly II 1 and 4), played an important role in the extensive Arabic work on the solution of cubic equations. This began in the tenth century with the work of Abu Ja'far al-Khazin, continued in the eleventh century with the work of Omar Kayyam , and culminated in the twelfth ,with Sharaf al-Din al-Tusi's masterly treatment of the problem. It also stimulated al-Kuhi's solution of the difficult problem of constructing a spherical segment whose surface is equal to that of one segment and whose volume is equal to that of another.

Apollonius

In the ninth century two men, Hilal b. Abi Hilal al-Himsi and Thabit ibn Qurra, translated the first seven books of Apollonius's great work, the *Conics*, under the patronage and active involvement of the Banu Musa. However, Islamic geometers knew notjust the *Conics* but virtually all of the Apollonius's works, many of them long since lost, and a number of geometers studied these works closely.

The *Conics* provided the basic theoretical material for the development of a geometrical theory of solution of cubic equations, as well as a basis for the solution of advanced geometric problems, such as those solved by al-Kuhi. An example of such a problem was one posed by Abu Sahl al-Kuhi: To construct a straight line passing through a given point P and cutting two straight lines m and m' as well as an arbitrary curve c so that the sections these three cut off from the line have a given ratio. He used a hyperbola to solve this problem, and he also used a hyperbola to construct an equilateral pentagon in a square such that only one of its sides lies on a side of the square.

The famous tenth-century scientist Ibn al-Haytham, who was from Basra in present-day Iraq and spent most of his adult life in Egypt, studied the *Conics* very deeply, sufficiently so that he felt able to attempt a restoration of the lost eighth book. His study of the *Conics* also made itself felt in his *Optics* and his treatise on burning mirrors. This latter treatise, together with the writings of Archimedes, was the main source for Western knowledge of conic sections prior to Commandino's important translation of the *Conics* from Greek in 1566.

Ptolemy

Surely, the two most influential scientific works translated into Arabic from Greek are Euclid's *Elements* and Ptolemy's *Almagest*. The trigonometry of the latter work -combined with the Hindu sine function- was the source for trigonometry in medieval Islam. *The Almagest* also furnished the fundamental models of planetary motion for Islamic astronomy, it showed how to use observations to determine the parameters of these models, and it showed how to use the models to construct tables predicting astronomical phenomena. These tables were the essential components of the Arabic astronomical handbooks (known as zijes).

Like the *Elements* the *Almagest* was translated into Arabic several times. The first was at the suggestion of the Yahya b. Khalid (738- 805) a member of the wealthy Barmakid family. He, however, was not satisfied with the translation and had it redone by a group of translators working under the Director of the House of Wisdom in Baghdad, who was evidently responsible for 'quality control'. According to the tenth-century writer, Ibn al-Nadim, 'it is said' that al-Hajjaj b. Matar, one of the translators of Euclid, also translated *The Almagest*, and Thabit corrected the old translation of the entire book. Then Ishaq ibn Hunayn translated it, with Thabit correcting that as well. In the end, though, Thabit's earlier correction was more successful. Also, like the *Elements*, the work gave rise to commentaries and corrections.

Unlike the *Elements*, however, it also gave rise to numerous works of the same intent and structure but which often incorporated a variety of new data, reflecting new observations, calculations, or information. It also stimulated highly interesting work in mathematical astronomy. One feature of the Ptolemaic models, which seemed to violate any physical sense, forced a sphere to rotate uniformly around a point not at its center. Muslim astronomers from the late 10th century onward expended considerable effort on eliminating this feature of Ptolemy's models, and in the end, they succeeded. The main work was done by a group of astronomers in Maragha, in present-day Iran, of whom the best known is the thirteenth-century mathematician, astronomer, and philosopher, Nasr al-Din al-Tusi. In particular Nasr al-Din solved the geometric problem of producing rectilinear motion from circular motion by a device now known as "the Tusi couple".

Another work of Ptolemy, his *Planispherium*, is the earliest known work on the astrolabe, an instrument which became both teacher and tool of medieval astronomers, whether in Islam or the Latin West.

Although the Greek original of Ptolemy's work has been lost, the work survives in Arabic and in Latin (from the Arabic). Medieval Islam also acquired a Greek treatise on the astrolabe by the sixth-century Greek writer, John Philoponos, and a Syriac treatise by Severus Sebokht, the abbot of a monastery on the upper Euphrates River. One of the earliest Arabic writers on the astrolabe was 'Ali ibn 'Isa who, in the early ninth century, explained its use for finding time during the day or night and for finding the direction of prayer (i.e. the direction of Mecca). Both of these were responses to the needs of Islamic society, and the problem of finding the direction of Mecca

led to an Islamic innovation in the design of this instrument, namely the addition of azimuth lines indicating directions on to the local horizon.
The instrument itself had a rich history in the Islamic world, and it developed into the astrolabe quadrant - which became such a favorite in Ottoman times. In the eleventh century the Spanish muslim astronomer, al-Zarqallu, invented an astrolabe which would work at any latitude. Later the Persian astronomer, Al-Muzaffar ibn Muzaffar al-Tusi, who died in the early thirteenth century, showed how, using only a graduated rod with a plumb bob and a cursor, one could perform all the functions of the astrolabe.
The astrolabe was, among other things, a chief source of timekeeping and astronomical instruction in the ancient and medieval worlds. Its introduction into the west also introduced the words "zenith", "azimuth" and "nadir" into western languages.
A third work of Ptolemy's that had a great influence in medieval Islam was his *Geography*. This work not only taught the principles of scientific map making but also furnished a data-base of latitudes and longitudes for about 8,000 places in the ancient world. An epitome of the *Geography* was translated into Syriac and the book was translated into Arabic first for the famous Arab philosopher, Abu Yusuf al-Kindi (801- 873). Like that of the *Almagest*, however, the first translation was not particularly good. Thabit ibn Qurra then retranslated it and did, as usual, a good job. However, no medieval Arabic translation of the *Geography* survives. The Arabic translation that does exist was done, on the order of the Ottoman sultan who conquered Constantinople in 1453. It testifies to a developing Ottoman interest in scientific geography, but it as too late to have any impact on the western tradition of that work.
Ptolemy's Geography had considerable impact on medieval Islam. In the early ninth-century, the same al-Khwarizmi whose writings introduced Hindu numeration to medieval Islam also corrected Ptolemy's length of the Mediterranean from 62° to 52° in his work *The Image of the Earth.* In the thirteenth century, the astronomer al-Marrakushi carried this further and corrected al-Khwarizmi's figure to 44° , a figure within two degrees of the correct 42° .
Correction of longitudes is, of course, only half of the problem of finding correct geographical coordinates for localities. In the late 10th to the early 11th century al-Biruni wrote his *Book on the Determination of Coordinates of Localities*, devoted to determining accurate longitudes and latitudes for

a chain of cities from Baghdad to Ghazna in present day Afghanistan, the capital of his patron Mahmud.

In another work, *Book of the Projection of the Constellations and Making Spheres Plane*, al-Biruni published new map grids, one known as Arrowsmith's projection and another known as Postel's projection. (For illustrations of these projections see Berggren 1982.)

Al-Biruni was by no means alone in working on the coordinates of localities. We mention here two other efforts: 1. A survey of Anatolia done in the 10th or 11th centuries, i.e. at a time roughly contemporary with al-Biruni, included over 450 places. 2. The astronomer al-Marrakushi, whom we mentioned above, much improved Ptolemy's longitudes for a number of localities in the Maghreb.

Conclusion

In this brief paper we have given an introduction to the reception of Greek geometry in medieval Islam. In the course of discussing the transmission of the works of Euclid, Archimedes, Apollonius and Ptolemy, we have pointed out the sometimes problematic status of Greek geometry, which came from pagans, in medieval Islam. We have also indicated some of the motives that medieval Islam had for acquiring this science. These included the utility of the subject for astronomy (and thereby also astrology) and geography, its relevance to problems of determining times of prayer and the direction of Mecca, and the intellectual fascination its problems had for Muslim mathematicians. Readers in the Spanish speaking world who want to acquaint themselves with the way the geometrical tradition developed in Islam should obtain the works of the Barcelona school. Its members, under the able leadership of Juan Vernet and Julio Samsó, have published a number of excellent studies of the history of mathematics in medieval Islam, particularly its astronomical aspects. A good place to begin, for an overview of Islamic science generally (though restricted to western Islam), would be *El Legado Cientifico Andalusí* (1992). From this book the reader could go on to any of the studies cited in the bibliographies of the essays that constitute that it. The Barcelona group also publishes a journal, *Suhayl*, which contains scholarly studies of all aspects of Islamic mathematics and astronomy.

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