*CUBO A Mathematical Journal*
*Vol.21, Nº 03, (93–105). December 2019*
http://dx.doi.org/10.4067/S0719-06462019000300093

# Wave propagation through a gap in a thin vertical wall in deep water

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## ABSTRACT

The problem of oblique scattering of surface water waves by a vertical wall with a gap submerged in infinitely deep water is re-investigated in this paper. It is formulated in terms of two first kind integral equations, one involving the difference of potential across the wetted part of the wall and the other involving the horizontal component of velocity across the gap. The integral equations are solved approximately using one-term Galerkin approximations involving constants multiplied by appropriate weight functions whose forms are dictated by the physics of the problem. This is in contrast with somewhat complicated but known solutions of corresponding deep water integral equations for the case of normal incidence, used earlier in the literature as one-term Galerkin approximation. Ultimately this leads to very closed (numerically) upper and lower bounds of the reflection and transmission coefficients so that their averages produce fairly accurate numerical estimates for these coefficients. Known numerical results for normal incidence and for a narrow gap obtained by other methods in the literature are recovered, thereby confirming the correctness of the method employed here.

**RESUMEN**

En este artículo re-investigamos el problema de dispersión oblicua de ondas superficiales de agua por una pared vertical con una abertura sumergida en agua infinitamente profunda. Se formula en términos de dos ecuaciones integrales de primera especie, una involucrando la diferencia de potencial a través de la parte mojada de la pared y la otra involucrando la componente horizontal de la velocidad a través de la apertura. Las ecuaciones integrales son resueltas aproximadamente usando aproximaciones de Galerkin de un término involucrando constantes multiplicadas por funciones peso apropiadas, cuyas formas son dictadas por la física del problema. Esto se contrapone con lo complicado de soluciones conocidas para las correspondientes ecuaciones integrales de agua profunda para el caso de incidencia normal, usadas anteriormente en la literatura como aproximaciones de Galerkin de un término. Últimamente esto lleva a cotas superiores e inferiores muy cercanas (numéricamente) para los coeficientes de reflexión y transmisión de tal suerte que sus promedios producen estimaciones numéricas razonablemente precisas para estos coeficientes. Se recuperan resultados numéricos conocidos en la literatura para la incidencia normal y para una apertura delgada, confirmando que los métodos empleados son correctos.

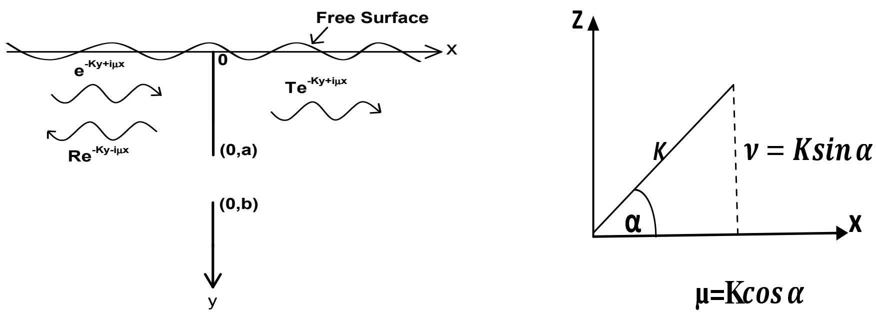
**Keywords and Phrases:** Thin vertical wall, submerged gap, integral equations, One-term Galerkin approximations, Constant as basis, Reflection and transmission coefficients.

**2010 AMS Mathematics Subject Classification:** 76B07, 76B15.

# 1 Introduction

The problem of oblique scattering of surface water waves by a thin vertical wall with a gap of arbitrary width submerged in infinitely deep water is re investigated here within the framework of linearized theory of water waves. Porter [10] investigated this problem for normal incidence of a surface wave train employing a reduction procedure and also an integral equation formulation, both leading to the same Riemann-Hilbert problem in the theory of complex variable, and the reflection and transmission coefficients are obtained in closed forms in terms of some definite integrals which could be computed numerically. When the gap is narrow, Tuck [12] earlier employed the method of matched asymptotic expansion to obtained the transmission coefficient approximately in terms of an analytical expression. Packham and Williams [9] employed an integral equation formulation based on Green's integral theorem to reduce the problem of narrow gap in uniform finite depth water to a first kind integral equation in horizontal component of velocity across the gap. They solved the integral equation approximately exploiting the concept of narrowness of the gap, and obtained an approximate analytical expression for the transmission coefficient. Mandal [7] employed an integral equation formulation based on Havelock's [6] expansion of water wave potential to solve the narrow gap problem in deep water for normal incidence, and obtained the transmission coefficients approximately by exploiting the concept of narrowness of the gap as has been done by Packham and Williams [9]. Chakrabarti et al [1] re-investigated Porter's problem by reducing it to a special logarithmic singular integral equation involving two unknown constants, one involving the unknown reflection coefficient, which were ultimately determined by two solvability criteria. Das et al [2] investigated the oblique scattering problem by formulating it in terms of two first kind integral equations after employing Havelock's [6] expansion of water wave potential, one involving the horizontal component of velocity across the gap and the other involving the difference of potential across the wetted parts of the wall. These were then solved approximately employing one-term Galerkin approximations involving somewhat complicated but exact solutions of the corresponding integral equations for the case of normal incidence as could be found from Porter [10]. Also, one-term Galerkin technique was employed recently by Roy et al [11] while studying the problem of water wave scattering by a pair of thin vertical barriers with unequal gaps submerged in deep water. However, it involves somewhat complicated but known exact solutions of the corresponding integral equations for a single barrier partially immersed in deep water and for normal incidence, as basis functions.

In the present paper, this problem is re-investigated employing one-term Galerkin approximation technique wherein the one-term approximations are taken to be simply constants multiplied by appropriate weight functions whose forms are dictated by the physics of the problem. This technique leads to very accurate close bounds(numerical) for the reflection and transmission coefficients so that their averages produce accurate numerical estimates for these coefficients. Known numerical results for normal incidence and also for a narrow gap obtained by other methods in the literature are recovered from the results obtained by the present method as special cases, thereby confirming the correctness of the method. Numerical results obtained by the present method are displayed graphically in a number of figures. It may be noted that this type of one-term Galerkin method to solve integral equations has not been employed in the literature on water waves earlier.




(a) Barrier configuration (b) Angle of incident wave

Figure 1: Sketch of the problem.

## 2 Mathematical formulation and solution

A Cartesian co-ordinate system is taken in which $y$-axis is chosen vertically downwards in the fluid region and the $x, z$-plane is taken as the rest position of the free surface. For a thin vertical wall with a gap submerged in deep water, its wetted parts are represented by $x = 0, y \in L = (0, a) \bigcup (b, \infty)$, wherein the gap is represented by $x = 0, y \in \bar{L} = (a, b)$. The problem is described in figure 1 wherein $R$ and $|T|$ denote the reflection and transmission coefficient respectively. Full details of the problem is given in Das et al.[2] . For the problem of oblique scattering of surface water waves by the wall with a gap, let $f(y)(y \in \bar{L})$ denote the horizontal component of velocity across the gap, $g(y)(y \in \bar{L})$ denote the difference of potential function across the wetted parts of the wall, $R$ and $T$ denote the reflection and transmission coefficients respectively. Then the behaviors of $f(y)$ and $g(y)$ at the end points $y = a, y = b$ are given by

$$f(y) = \begin{cases} O\left((y-a)^{-\frac{1}{2}}\right) & \text{as } y \to a+0, \\ O\left((b-y)^{-\frac{1}{2}}\right) & \text{as } y \to b-0, \end{cases} \tag{2.1a}$$

and

$$g(y) = \begin{cases} O\left((a-y)^{\frac{1}{2}}\right) & \text{as } y \to a-0, \\ O\left((y-b)^{\frac{1}{2}}\right) & \text{as } y \to b+0. \end{cases} \tag{2.1b}$$

The relation between $R, T$ and $f(y), g(y)$ are given by

$$T = 1 - R = -2i\sec\alpha \int_{\bar{L}} f(y)e^{-Ky}\,dy, \tag{2.2a}$$

$$R = -K\int_{L} g(y)e^{-Ky}\,dy, \tag{2.2b}$$

where $\alpha$ is the angle of incidence of train of surface water waves on the thin wall, $K = \frac{\sigma^2}{g}$ , $\sigma$ being the angular frequency and $g$ is the gravity.

Let

$$F(y) = -\frac{2}{\pi R}f(y), y \in \bar{L}, \tag{2.3a}$$

$$G(y) = \frac{1}{\pi i K\cos\alpha(1-R)}g(y), y \in L, \tag{2.3b}$$

then it is easy to see that $G(y)$ and $F(y)$ satisfy the first kind integral equations (cf. Das et al [2], Mandal and Chakrabarti [8])

$$(\mathcal{M}G)(y) \equiv \int_L G(u)\mathcal{M}(y,u)du = e^{-Ky}, y \in L \tag{2.4a}$$

and

$$(\mathcal{N}F)(y) \equiv \int_L F(u)\mathcal{N}(y,u)du = e^{-Ky}, y \in \bar{L} \tag{2.4b}$$

where

$$\mathcal{M}(y,u) = \lim_{\epsilon\to+0}\int_0^\infty \frac{k_1 S(k,y)S(k,u)}{k^2+K^2}e^{-\epsilon k}dk, \tag{2.5a}$$

and

$$\mathcal{N}(y,u) = \int_0^\infty \frac{S(k,y)S(k,u)}{k_1(k^2+K^2)}dk, \tag{2.5b}$$

where $k_1 = \left(k^2+\nu^2\right)^{\frac{1}{2}}, \nu = K\sin\alpha, S(k,y) = k\cos ky - K\sin ky$ while (2.2a) and (2.2b) produce

$$\int_{\bar{L}} F(y)e^{-Ky}\,dy = C, \tag{2.6a}$$

$$\int_L G(y)e^{-Ky}\,dy = \frac{1}{\pi^2K^2C} \tag{2.6b}$$

where

$$C = \frac{1-R}{i\pi R}\cos\alpha. \tag{2.7}$$

(2.5a) and (2.5b) show that $\mathcal{M}(y,u)$ and $\mathcal{N}(y,u)$ are real and symmetric so that $G(u), F(u)$ satisfying (2.4a) and (2.4b) respectively are real and hence, $C$ satisfying (2.6a) as well as (2.6b), is an unknown real quantity. Once $C$ is found $R$ and $T(=1-R)$ can be calculated using (2.7).

If $G(y)$ and $F(y)$ are chosen as one-term Galerkin approximations given by

$$G(y) \approx c_0 g_0(y), y \in L; F(y) \approx d_0 f_0(y), y \in \bar{L}, \tag{2.8}$$

then exploiting the properties of symmetry, self-adjointness and positive semi-definiteness of the integral operators $(\mathcal{M}G)(y)$ and $(\mathcal{N}f)(y)$ defined by (2.4) proceeding as in Evans and Morris [4] and Das et al [2], it can be shown that $C$ has the bounds $A, B$

$$B \leq C \leq A \tag{2.9}$$

where $A$ and $B$ can be expressed in terms of integrals involving $g_0(y)$ and $f_0(y)$ respectively as given by

$$A = \frac{1}{\pi^2 K^2} \frac{\int_L g_0(y)(\mathcal{M}g_0)(y)dy}{(\int_L g_0(y)e^{-Ky}dy)^2}, \tag{2.10}$$

$$B = \frac{(\int_{\bar{L}} f_0(y)e^{-Ky}dy)^2}{\int_{\bar{L}} f_0(y)(\mathcal{N}f)(y)dy}. \tag{2.11}$$

It may be noted that $A, B$ are independent of $c_0, d_0$ so that these can be chosen to be unity. The upper and lower bounds for $|R|$ and $|T|$ are now obtained as

$$R_1 \leq |R| \leq R_2, T_1 \leq |T| \leq T_2 \tag{2.12}$$

where

$$R_1 = \frac{1}{(1+\pi^2 A^2 \sec^2\alpha)^{\frac{1}{2}}}, R_2 = \frac{1}{(1+\pi^2 B^2 \sec^2\alpha)^{\frac{1}{2}}}, \tag{2.13a}$$

$$T_1 = \frac{\pi B \sec\alpha}{(1+\pi^2 A^2 \sec^2\alpha)^{\frac{1}{2}}}, T_2 = \frac{\pi A \sec\alpha}{(1+\pi^2 B^2 \sec^2\alpha)^{\frac{1}{2}}}. \tag{2.13b}$$

Das et al [2] chose $g_0(y)$ and $f_0(y)$ as the exact solutions of the integral equations (2.4a) and (2.4b) for the case of normal incidence($\alpha = 0^0$) and these involve quite complicated expressions (cf. Mandal and Chakrabarti [8]). Here we choose $g_0(y), f_0(y)$ as

$$g_0(y) = \begin{cases} \left(1-\frac{y}{a}\right)^{\frac{1}{2}}, 0 < y < a, \\ \\ e^{-Ky}\left(\frac{y}{b}-1\right)^{\frac{1}{2}}, b < y < \infty \end{cases} \tag{2.14a}$$

and

$$f_0(y) = \frac{a}{\{(y-a)(b-y)\}^{\frac{1}{2}}}, a < y < b. \tag{2.14b}$$

This choice of $f_0(y)$ and $g_0(y)$ is dictated by the behaviors of $f(y)$ and $g(y)$ at the end points $y = a$ and $y = b$.

Then, after substituting (2.14a) in (2.10), $A$ is obtained as

$$A = \frac{\int_0^\infty \frac{k_1}{k^2+K^2}\left[kU(a,b,k,K) - KV(a,b,k,K)\right]^2 dk}{\pi^2 K^2 W^2(a,b,k,K)} \tag{2.15}$$

where

$$U(a,b,k,K) = \int_0^a \left(1-\frac{y}{a}\right)^{\frac{1}{2}} \cos ky\, dy + \int_b^\infty e^{-Ky}\left(\frac{y}{b}-1\right)^{\frac{1}{2}} \cos ky\, dy,$$

$$V(a,b,k,K) = \int_0^a \left(1-\frac{y}{a}\right)^{\frac{1}{2}} \sin ky\, dy + \int_b^\infty e^{-Ky}\left(\frac{y}{b}-1\right)^{\frac{1}{2}} \sin ky\, dy,$$

$$W(a,b,K) = \int_0^a e^{-Ky}\left(1-\frac{y}{a}\right)^{\frac{1}{2}} dy + \int_b^\infty e^{-2Ky}\left(\frac{y}{b}-1\right)^{\frac{1}{2}} dy.$$

$U(a,b,k,K), V(a,b,k,K)$ and $W(a,b,K)$ can be expressed analytically in terms of Young's and lower incomplete gamma functions(cf. Gradshteyn and Ryzhik [5]).

Similarly, after substituting (2.14b) in (2.11), $B$ is obtained as

$$B = \frac{M_{0,0}^2(K(b-a))e^{-K(a+b)}}{K(b-a)\int_0^\infty \frac{J_0^2(\frac{k(b-a)}{2})}{k_1(k^2+K^2)}\left[k\cos k(\frac{a+b}{2}) - K\sin k(\frac{a+b}{2})\right]^2 dk} \tag{2.16}$$

where $M_{0,0}$ is the Whittaker function and $J_0$ is the Bessel function.

## 3 Numerical results

The lower and upper bounds of the reflection and transmission coefficients $|R|$ and $|T|$ respectively are evaluated numerically for various values of different parameters such as wavenumber $Kb$, angle of incidence $\alpha$ and $\frac{a}{b} = 0.5$. Only the lower and upper bounds $R_1$ and $R_2$ of $|R|$ are displayed in Table 1. Here we put $\alpha = 0^0$ in the expressions for $R_1$ and $R_2$ for obtaining numerical estimates for $|R|$ for the case of normal incidence and the bounds are also compared with exact values derived from Porter's [10] exact analytical results. Numerical values of upper and lower bounds of $|R|$ coincide within 3 to 4 decimal places and hence their averages provide very accurate estimates for the reflection coefficients. Similar computations have been carried out for the upper and lower bounds of $|T|$. However, these results are not displayed here. It has also been checked that these numerical estimates satisfy the energy identity $|R|^2 + |T|^2 = 1$, which provides a partial check on the correctness of the method. There are also other checks as described below. Also the numerical results presented in Table 1 are compared with those in Table 3 of Das et al [2]. Almost the same results are obtained. It may be noted that for the present method, the basis function $g_0(y)$ given by (2.14a) decays exponentially as $y \to \infty$ while for the method employed in Das et al [2] the basis function $f_1(y)$ given by (5.2) (and (5.3)) of Das et al [2] decays algebraically as $y \to \infty$. Because of this, the one-term Galerkin method with simplified basis functions employed here provides high accuracy in the numerical results.

| | $\alpha = 0^0$ | | | $\alpha = 30^0$ | | $\alpha = 60^0$ | | $\alpha = 85^0$ | |
|---|---|---|---|---|---|---|---|---|---|
| $Kb$ | $R_1$ | $R_2$ | $\lvert R\rvert$ Porter[1] | $R_1$ | $R_2$ | $R_1$ | $R_2$ | $R_1$ | $R_2$ |
| 0.05 | 0.7251 | 0.7257 | 0.7251 | 0.6582 | 0.6587 | 0.4106 | 0.4109 | 0.0831 | 0.0831 |
| 0.4 | 0.4343 | 0.4344 | 0.4343 | 0.3605 | 0.3625 | 0.1823 | 0.1875 | 0.0306 | 0.0307 |
| 1.2 | 0.6500 | 0.6504 | 0.6502 | 0.5872 | 0.5877 | 0.3752 | 0.3755 | 0.0733 | 0.0772 |
| 2.0 | 0.9448 | 0.9472 | 0.9466 | 0.9236 | 0.9238 | 0.7950 | 0.7954 | 0.2092 | 0.2099 |
| 3.0 | 0.9960 | 0.9987 | 0.9960 | 0.9936 | 0.9937 | 0.9725 | 0.9771 | 0.6100 | 0.6107 |
| 4.0 | 0.9996 | 0.9999 | 0.9996 | 0.9993 | 0.9994 | 0.9967 | 0.9969 | 0.9206 | 0.9206 |

Table 1. Lower and upper bounds for the reflection coefficient of $|R|$ for various values of the parameters $Kb$, $\alpha$ and $\frac{a}{b} = 0.5$

As in Porter [10] and Tuck [12], let $a = h(1 - \frac{\mu}{2}), b = h(1 + \frac{\mu}{2}), \lambda = \frac{2\pi}{K}$ where $h$ is the depth of the center of the gap below the free surface , $\mu$ is the ratio of the width of the gap to its mean depth and it lies between 0 to 2 and $\lambda$ is the wavelength of the incident wave.

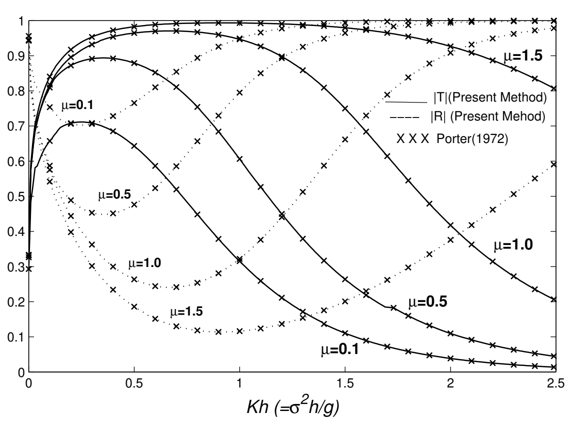



Figure 2: $|R|(...)$ and $|T|(-)$ against $Kh$ for different values of $\mu$, and $\alpha = 0^0$.

In figure 2, $|R|$ and $|T|$ are depicted against $Kh(= \frac{K(a+b)}{2})$ for different values of $\mu(= \frac{2(b-a)}{b+a})$

and for normal incidence($\alpha = 0^0$). Also $|R|$ and $|T|$ calculated from Porter's [1] exact expressions obtained by a completely different method are indicated in figure 2 by cross marks (x). From this figure it is observed that the curves of $|R|$ and $|T|$ plotted on the basis of the numerical results obtained by the present method and plotted on the basis of Porter's [10] exact results coincide. This gives another check on the correctness of the method.

In figure 3, $|T|^2$ is depicted against $\frac{h}{\lambda}(=\frac{K(a+b)}{4\pi})$ for different small values of $\mu = 0.05, 0.15, 0.4$ and for normal incidence($\alpha = 0^0$) so that the gap is narrow. Also $|T^2|$ calculated from Tuck's [12] result (expression given in (6.2) there) are indicated in figure 3 by cross marks (x). From this figure it is observed that the curves of $|T^2|$ plotted on the basis of the numerical results obtained by the present method and plotted on the basis of Tuck's [12] approximate result obtained by the method of matched asymptotic expansion coincide. This provides yet another check for the correctness of the results obtained by the present method.

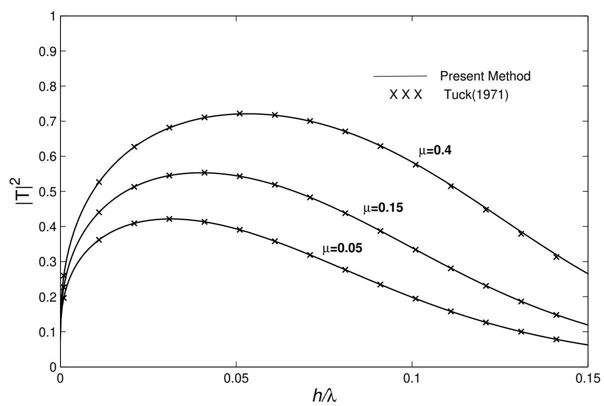



Figure 3: $|T|^2$ against $\frac{h}{\lambda}$ for different values of $\mu$, and $\alpha = 0^0$.

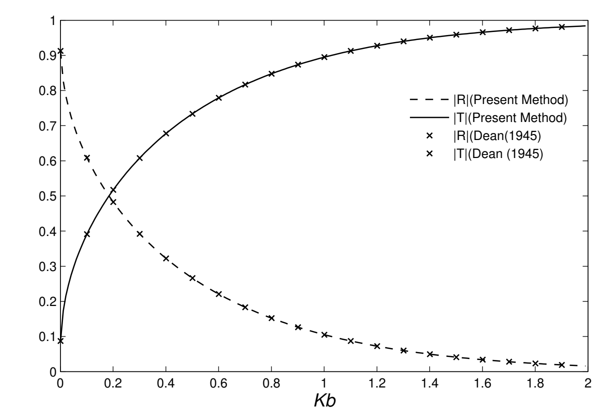


Figure 4: $|R|(...)$ and $|T|(-)$ against $Kb$ for $\alpha = 0^0$.

In figure 4, $|R|$ and $|T|$ are depicted graphically against the wavenumber $Kb$ for $\frac{a}{b} = 0$ so that the upper part of the wall is absent and the wall becomes a submerged barrier considered by Dean [3]. The curves of $|R|$ and $|T|$ almost coincide with the corresponding curves given by Dean [12] (indicated here by cross (x) marks). This produces a final check for the correctness of the results obtained by the present method.

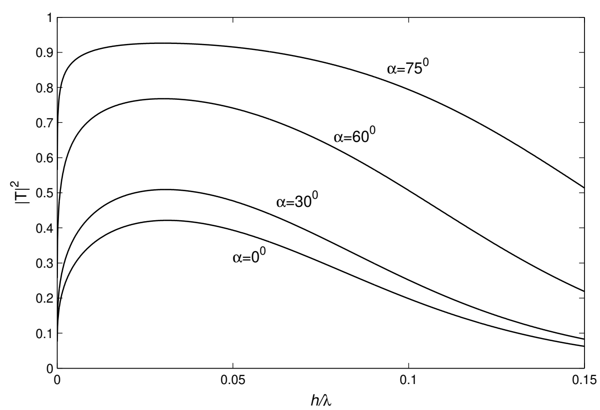



Figure 5: $|T|^2$ against $\frac{h}{\lambda}$ for different values of $\alpha$, and $\mu = 0.05$

In figure 5, $|T|^2$ is depicted against $\frac{h}{\lambda}$ for different values of $\alpha$ with fixed $\mu = 0.05$(narrow gap). This is in fact an extension of Tuck's figure for a narrow gap and normal incidence to oblique incidence. All the conclusion drawn by Tuck [12] for normal incidence about the transmission of energy through a narrow gap can be extended for oblique incidence. For example, considerable transmission of energy occurs for long waves. From the figure 5 it is observed that transmission increases with the increase in the angle of incidence which is plausible. Also for a fixed angle of incidence, transmission first increases as the wavenumber increases and then it decreases steadily as the wavenumber further increases. This is due to the fact that for large wavenumber the waves are confined near the free surface so that most of these are reflected by the upper part of the thin wall.

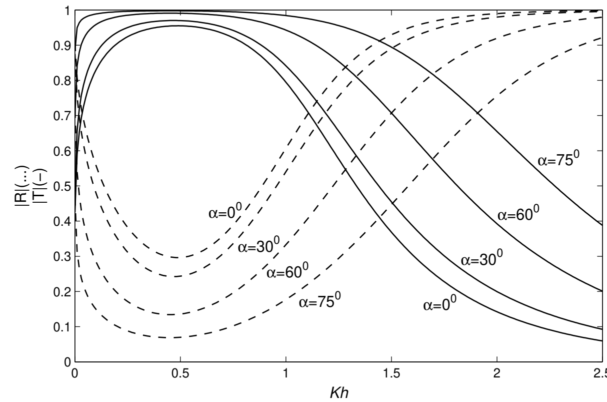



Figure 6: $|R|(...)$ and $|T|(-)$ against $Kh$ for different values of $\alpha$, and $\mu = 0.5$

In figure 6, $|R|$ and $|T|$ are depicted against $Kh$ for different values of $\alpha$ with fixed $\mu = 0.5$(moderate gap). This is again an extension of Porter's [10] curves for oblique incidence. This figure shows that for a wall with a moderate gap, as the angle of incidence increases, reflection coefficient decreases while transmission increases for fixed wavenumber. Incident waves are reflected by two parts of the wall. Obviously this reflection is maximum when waves are incident normally ($\alpha = 0^0$) on the wall and then reflection decreases gradually as $\alpha$ increases. This is plausible from physical considerations. Here however results for values of $\alpha$ from $0^0$ to $75^0$ are presented.

Again for fixed angle of incidence the reflection coefficient first decreases with increase of wavenumber and then increases asymptotically to unity as the wavenumber further increases. This is also plausible since for large wavenumber, the waves are confined near the free surface as mentioned earlier, so that most of the incident waves are reflected back. Reverse of this happens for the transmission coefficient i.e, transmission increases with the increase in the angle of incidence and for a fixed angle of incidence, transmission increases first with the increase of wavenumber and then decreases steadily to zero as the wavenumber further increases. It is interesting to note that for fixed angle of incidence, $\mu(0 < \mu < 2)$ is a crucial parameter in determining the transmission of wave energy through the gap at certain wavelengths. For $\mu = 1.0$, $|T|$ attains maximum near $Kh = 0.5$ corresponding to about more than $90$ percent of wave energy transmission. For fixed $\alpha$, as $\mu$ decreases i.e, as gap becomes smaller, $|T|$ decreases for all finite $Kh$ which is shown in the figure 2. The curves in figures 5 and 6 may be regarded as new results.

# 4 Conclusion

The problem of water wave scattering by a thin vertical wall with a gap submerged in infinitely deep water is re-investigated by using integral equation formulations based on Havelock's expansion of water wave potential. Two first kind integral equations involving horizontal component of velocity across the gap and difference of velocity potential across the upper and lower parts of the wall are obtained. These are solved here approximately by using one-term Galerkin approximations involving constants multiplied by appropriate weight functions whose forms are dictated by the behaviour at the end points of the gap and at infinite depth. Exploitation of the symmetry and positive semi-definiteness of the operators of the integral equations lead to expressions for upper and lower bounds for the reflection and transmission coefficients. These bounds, when computed numerically, coincide upto 3-4 decimal places so that their averages produce very accurate numerical estimates for the reflection and transmission coefficients. Known numerical results(in the form of graphs) for the problem of water wave scattering by a thin wall with a gap, available in the literature by employing different methods, are recovered from the results obtained by the present method as special cases. The method employed here appears to be quite simple in comparison to other known methods employed for this problem. It is felt that this type of one-term Galerkin technique involving simple basis functions can be employed to study wave scattering by other types of obstacles with submerged edges such as multiple thin vertical barriers, thick rectangular barriers, wave scattering by step-type bottom topography etc.

## 5 Acknowledgments

The authors thank the referee for his comments and suggestions to improve the paper in the present form. B C Das thanks the UGC, India, for providing financial support (File no: 22/12/2013(ii)EU-V), as a research scholar of the University of Calcutta, India. This work is also supported by SERB through the research project no. EMR/2016/005315

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